Government of **Western Australia**
School Curriculum and Standards Authority

ATAR course examination, 2020

Question/Answer booklet

MATHEMATICS SPECIALIST

Place one of your candidate identification labels in this box.

Ensure the label is straight and within the lines of this box.

Section Two: Calculator-assumed

WA student number: In figures

In words

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Number of additional answer booklets used (if applicable):

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators, which can include scientific, graphic and Computer Algebra System (CAS) calculators, are permitted in this ATAR course examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	49	35
Section Two: Calculator-assumed	13	13	100	86	65
				Total	100

Instructions to candidates

1. The rules for the conduct of the Western Australian external examinations are detailed in the *Year 12 Information Handbook 2020: Part II Examinations*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific questions asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

See next page

Section Two: Calculator-assumed **65% (86 Marks)**

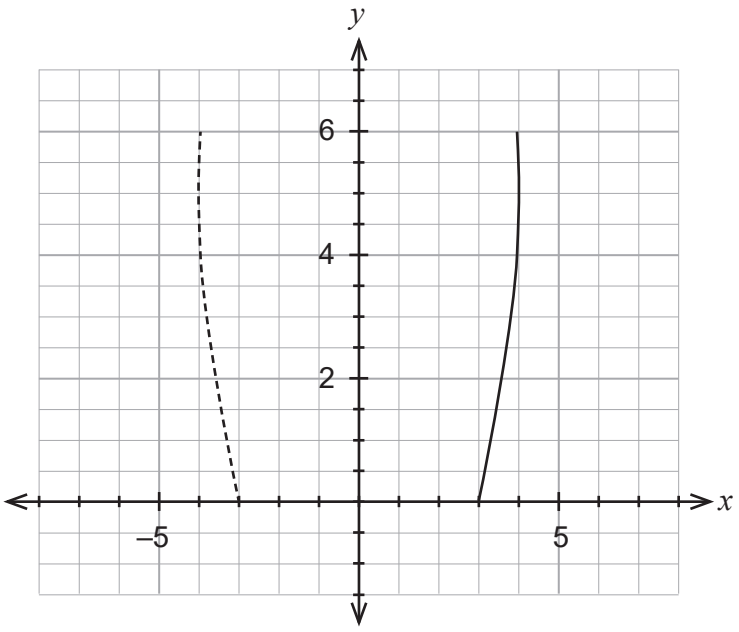
This section has **13** questions. Answer all questions. Write your answers in the spaces provided.

Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

Working time: 100 minutes.

Question 9 **(4 marks)**

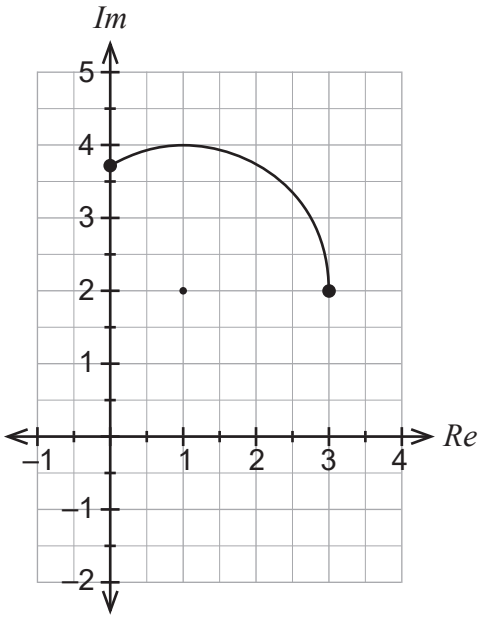
The shape of a small wine glass is modelled by revolving the curve $\sin\left(\frac{y}{\pi}\right) = x - 3$ about the y axis, where $0 \leq y \leq 6$. All dimensions are in centimetres.



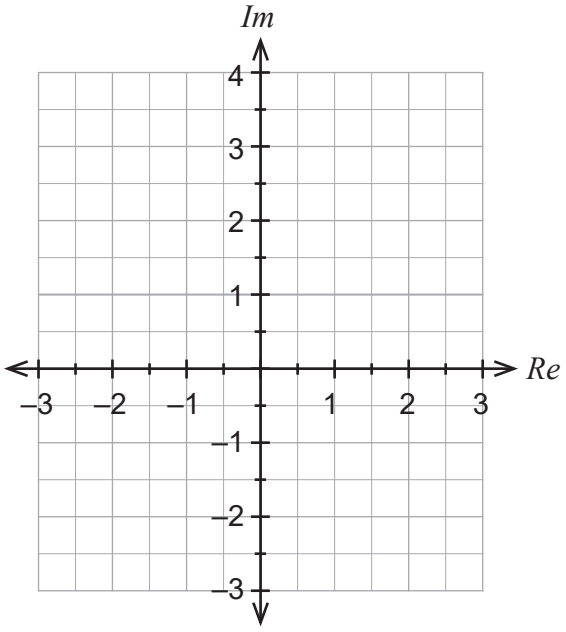
Calculate, correct to the nearest 0.01 cm, the depth of wine in the glass if it is to contain 80% of its maximum volume.

Question 10 **(7 marks)**

(a) The sketch of the locus of a complex number z has been shown below. Write equations or inequalities in terms of z (without using $x = Re(z)$ or $y = Im(z)$) for the indicated locus.
(4 marks)



(b) Sketch the locus of the equation $|z + 2| = |z - i| + \sqrt{5}$ in the Argand diagram below.
(3 marks)



See next page

Question 11 **(5 marks)**

Let z, w and u be complex numbers where:

$$w = (1 + i)\overline{z} \qquad Arg(w) = \frac{\pi}{3} \qquad |w| = 2$$

$$u = \frac{z}{2 - 2i}$$

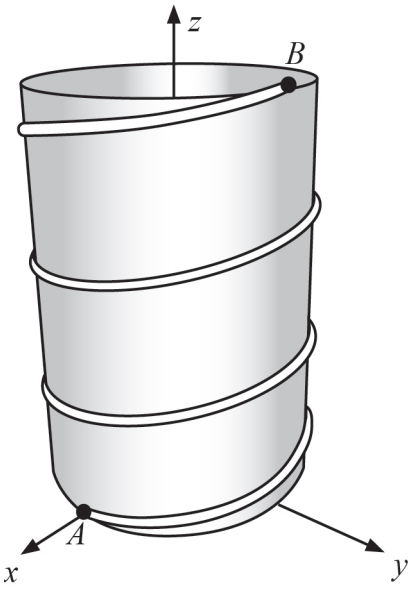
(a) Determine $Arg(u)$ exactly. (3 marks)

(b) Determine $|u|$ exactly. (2 marks)

See next page

Question 12 **(6 marks)**

A cylindrical shaped tower has a path that spirals upwards from the ground to an observation deck at point B as shown in the diagram below. The path begins at point A on the ground and finishes at point B at the top.



Let t = time in seconds that a tourist has been walking along the spiral path. The tourist takes 65π seconds to reach point B.

The tourist's position on this path at any time t is given by:

$$\underset{\sim}{r}(t) = \begin{pmatrix} 10\cos(0.1t) \\ 10\sin(0.1t) \\ 0.2t \end{pmatrix} \text{ metres.}$$

(a) Determine the height of the observation deck above the ground, correct to the nearest 0.01 metres. (1 mark)

(b) Determine the tourist's velocity $\underset{\sim}{v}(t)$. (2 marks)

See next page

(c) Show that the tourist walks at a constant speed and determine this speed, correct to 0.01 metres per second. (3 marks)

See next page

Question 13 **(4 marks)**

Solve the equation $z^4 = 8\sqrt{3} + 8i$ giving exact solutions in the form $r\,cis\,\theta$ where $-\pi < \theta \leq \pi$.

See next page

Question 14 **(5 marks)**

A particle travels in a straight line so that its velocity v cm per second and displacement x cm are related by the equation:

$$v = -0.2x$$

(a) Determine the acceleration a in terms of its displacement x. (2 marks)

(b) Does the particle's motion constitute simple harmonic motion? Justify your answer. (1 mark)

It is known that the initial displacement of the particle is $x = 4$ cm.

(c) Determine, correct to the nearest 0.01 second, when the particle has a displacement of 2 cm. (2 marks)

See next page

Question 15 **(6 marks)**

Let $z = r\,cis\,\theta$ be a complex number such that $\frac{\pi}{2} < \theta < \pi$.

(a) Express in terms of r and θ the complex number $\frac{\overline{z}}{-\sqrt{2}\,(i+1)}$. (3 marks)

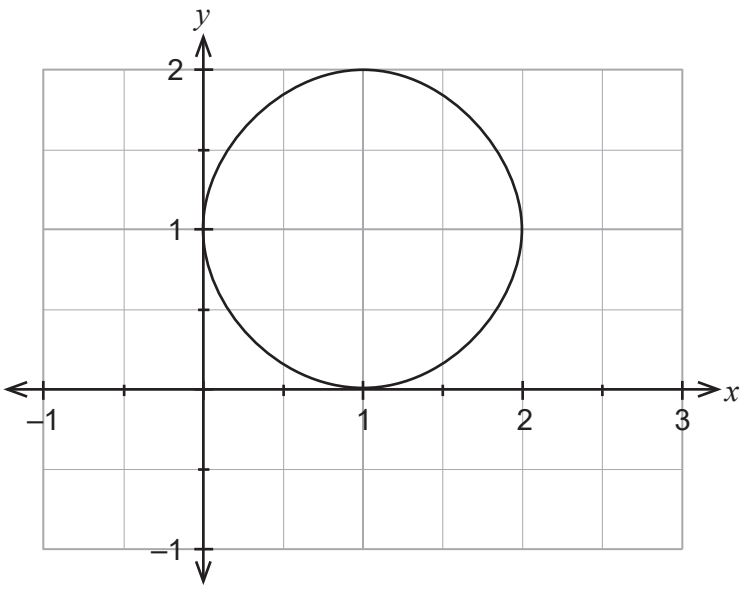
See next page

(b) Express $\alpha = Arg(z - ri)$ in terms of θ where $0 < \alpha < 2\pi$. (3 marks)

See next page

Question 16 **(5 marks)**

The curve shown below is given by the equation $(y-1)^2 = \sin\left(\frac{\pi x}{2}\right)$.



(a) Calculate the area, correct to 0.0001 square units, of the region that forms the interior of this curve. (3 marks)

(b) By using the answer to part (a), determine whether this curve is a circle. Explain your answer. (2 marks)

See next page

Question 17 **(8 marks)**

Members of a random sample of n shoppers at the El Cheepo shopping centre were asked by a consumer researcher how much they had spent in the shopping centre that day. Let μ denote the mean and σ the standard deviation of the amount spent. The standard deviation σ is known from previous research.

A 95% confidence interval for μ based on the sample is $150 \leq \mu \leq 200$ dollars.

(a) Determine the sample mean for this sample. (1 mark)

(b) Based on this confidence interval, calculate the standard deviation of the sample mean, correct to 0.01. (3 marks)

The following week, the researcher again took a random sample of shoppers from the El Cheepo shopping centre, but this time the sample size was doubled.

(c) What is the probability that the difference between μ and the sample mean from this sample will be less than \$10? (4 marks)

See next page

Question 18 **(11 marks)**

The mass of chocolate that is placed into each biscuit produced by the BikkiesAreUs company has been observed to be normally distributed with mean $\mu = 7.5$ grams and standard deviation $\sigma = 1.5$ grams.

(a) Determine the probability, correct to 0.01, that the total amount of chocolate used for 50 biscuits is less than 365 grams. (4 marks)

(b) If the probability that the mean amount of chocolate used per biscuit differs from μ by less than 0.2 grams is 98%, determine n, the number of biscuits that need to be sampled. (3 marks)

See next page

A competitor company called YouBeautChokkies produces similar biscuits. A sample of 144 biscuits was taken and it was found that the standard deviation of the mass of chocolate used in each biscuit was 1.8 grams and the total amount of chocolate used in the sample of 144 biscuits was 1.09 kg.

Charlie Chokka, a representative from the YouBeautChokkies company, stated that "we are using significantly more chocolate for each biscuit than BikkiesAreUs. If you want that real chocolate taste, then buy from us!"

(c) Perform the necessary calculations to comment on Charlie's claim. (4 marks)

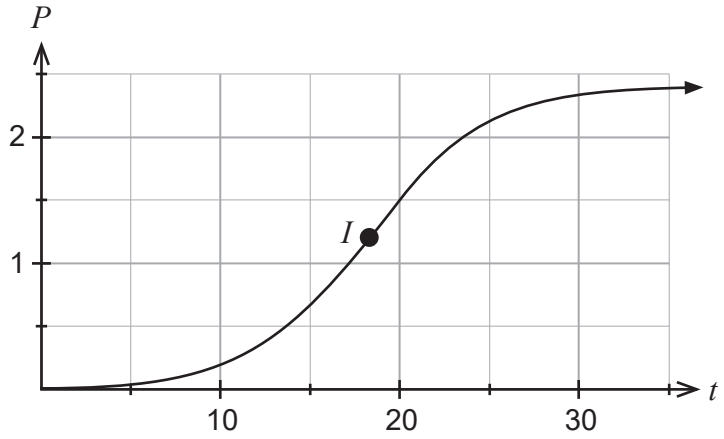
See next page

Question 19 **(11 marks)**

The population $P(t)$ of sardines in an ocean, measured in million tonnes after t years, was modelled by the logistic equation:

$$P(t) = \frac{2.4}{1 + 239e^{-0.3t}}$$

The graph of this model is shown below. This graph contains a point of inflection at point I.



(a) Calculate the size of the sardine ocean population at $t = 0$. (2 marks)

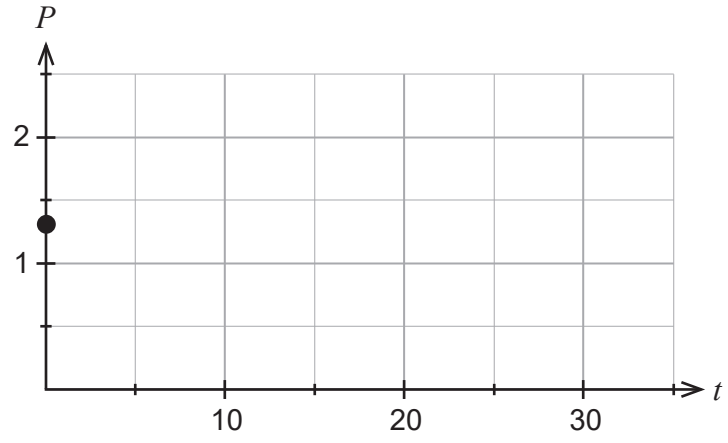
(b) Rewrite the logistic equation in the form $\frac{dP}{dt} = rP(k - P)$, stating clearly the values for r and k. (2 marks)

(c) When the sardine population is 500 000 tonnes, use the technique of increments to calculate the approximate change in population in the next month. (3 marks)

(d) Determine the maximum rate of growth of the sardine population. (2 marks)

Suppose that the initial population of sardines was 1.3 million tonnes.

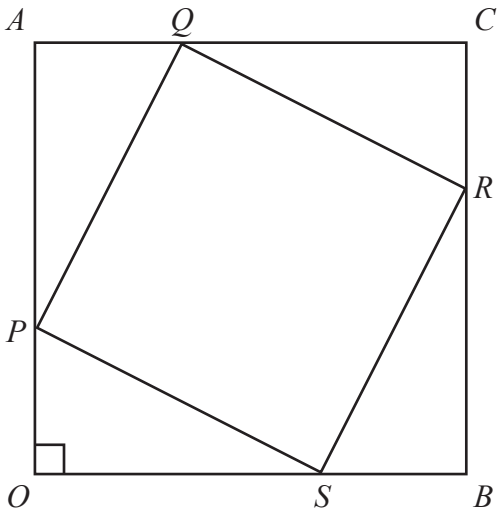
(e) Assuming that the rate of growth is still given by $\frac{dP}{dt} = rP(k - P)$ sketch the graph of the population growth on the axes below. Explain your graph. (2 marks)



Question 20 **(9 marks)**

Consider square $OACB$ where point O is the origin. Let the position vectors for points A, B be defined as $\underset{\sim}{a}, \underset{\sim}{b}$ respectively i.e. $\overrightarrow{OA} = \underset{\sim}{a}$ and $\overrightarrow{OB} = \underset{\sim}{b}$.

Let points P, Q, R and S be defined so that $\overrightarrow{OP} = k\underset{\sim}{a}$, $\overrightarrow{AQ} = k\underset{\sim}{b}$, $\overrightarrow{RC} = k\underset{\sim}{a}$ and $\overrightarrow{SB} = k\underset{\sim}{b}$ where $0 \leq k \leq 1$. This means that points P, Q, R and S are positioned along their respective sides in equal proportion.



(a) Using vector methods, prove that the size of $\angle PQR = 90°$. (5 marks)

See next page

Now suppose that in square $OACB$, it is known that $OA = 10$ cm and that point P is moving away from the origin at a speed of 0.2 cm per second. This means that points Q, R and S are moving at the same speeds along their respective sides.

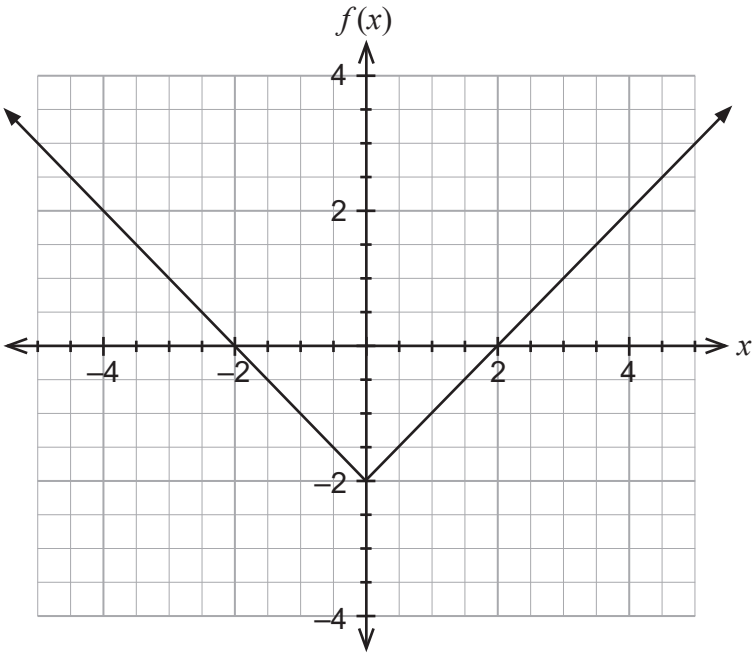
Let x = the distance OP.

(b) Determine the rate at which the area of square $PQRS$ is changing when $x = 3$ cm.

(4 marks)

Question 21 **(5 marks)**

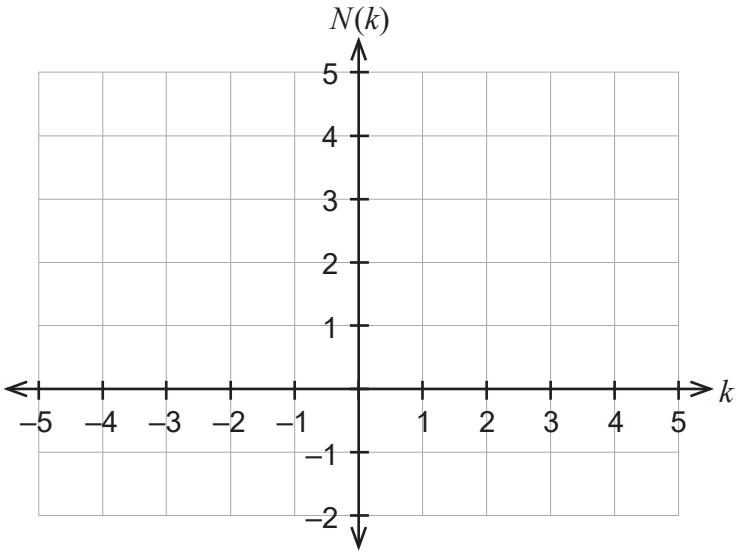
The sketch of the graph of $y = f(x)$ is shown below.



Consider the equation $|f(x)| = k$ where k is any real constant.

Define function $N(k)$ = the number of real solutions to the equation $|f(x)| = k$.

Sketch the graph of function $N(k)$ on the axes below.



A spare grid is provided at the end of this Question/Answer booklet. If you need to use it, cross out this attempt and indicate that you have redrawn it on the spare grid.

End of questions

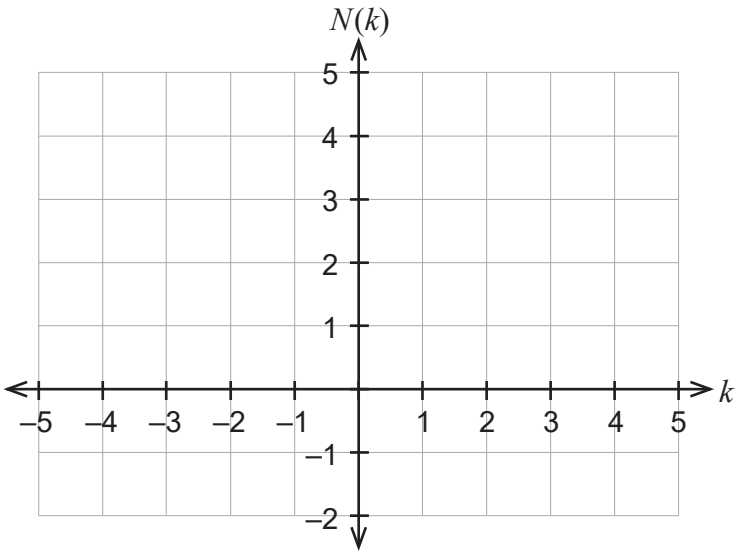
Supplementary page

Question number: _____

Supplementary page

Question number: _____

Spare grid for Question 21.



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